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SUBJECT: Skylab Wobble State in an Artificial
Gravity Experiment - Case 620

DATE: August 18, 1970

FROM: G. M. Anderson

ABSTRACT

Crew motion and other disturbances can perturb, or wobble, the steady rotation desired of the Skylab vehicle in an artificial gravity mode. The wobble state is characterized in this memorandum by describing the motion of the angular velocity vector and the vehicle principal axes with respect to the system angular momentum. A complimentary description of the motion of the angular velocity vector in vehicle coordinates is also given.

A parameter, the excess energy ratio, is suggested for use in characterizing the wobble state. The advantage of this parameter is its time invariance. The wobble angle parameters usually employed do not enjoy this important property.

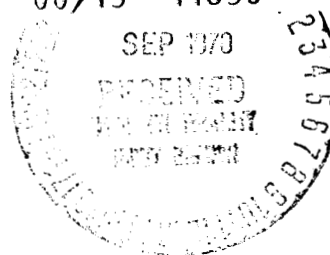
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MEMORANDUM FOR FILE

Introduction

An artificial gravity experiment is one objective being studied for Skylab B. The artificial gravity environment would be provided by rotating the vehicle about an axis perpendicular to the longitudinal axis of the vehicle. For passive stability the rotation must be about the axis of maximum moment of inertia.

Ideally the system angular momentum, angular velocity and axis of maximum moment of inertia are colinear. In this case the angular velocity is constant. Crew displacements and/or undesired torquing can disturb this condition and produce a state of variable angular velocity. If the incremental angular velocity introduced by the disturbance is small in magnitude relative to the desired steady value, the vehicle is said to wobble.

Wobble can produce physiological effects on the crew that may require corrective action. Whether or not wobble control is required depends on the acceptable wobble limit and the anticipated disturbances.

This memorandum describes the wobble state of the Skylab configuration. The interrelations of angular momentum, angular velocity and vehicle coordinates are given in both vehicle and inertial coordinates. A parameter, the excess energy ratio, is suggested for characterizing the wobble state. This parameter is superior to the usual characterization based on wobble angles since it is an invariant of the motion while wobble angles are in general time dependent.

The behavior of the system may be described in terms of elementary motions that are repeated in time. For inertial coordinates, insight into this regularity is provided by the Poincot construction. Changing the excess energy ratio merely scales the describing diagrams so that only two diagrams, one for vehicle space and one for inertial space are required.

The analysis is largely confined to the Appendix. Notation is given there. The treatment is confined to the rigid body model in a torque free environment. For the angular momentum levels anticipated, environmental disturbances have small effect in a time of the order of one orbit and this representation provides a good first level description of the rotational state.

Approximate Method

A great deal of information on the rotational characteristics of the Skylab can be obtained from an approximate solution of Euler's equations. These equations, which apply in vehicle principal axes coordinates,

$$I_x \dot{\omega}_x + (I_z - I_y) \omega_y \omega_z = 0$$

$$(1) \quad I_y \dot{\omega}_y + (I_x - I_z) \omega_z \omega_x = 0$$

$$I_z \dot{\omega}_z + (I_y - I_x) \omega_x \omega_y = 0$$

are solved taking

$$(2) \quad \omega_z \approx \omega_0 = \text{constant},$$

an assumption which linearizes the equations in ω_x and ω_y . The assumption is quite good. It will be shown later that the variation in ω_z does not exceed 1.3% for a rather large value, .001, of the excess energy ratio γ .^{*} For smaller values of γ the variation in ω_z is even less than the given value.

The solution of Equations (1) is

$$(3) \quad \omega_x = \omega_{x0} \cos \beta t - (a/b) \omega_{y0} \sin \beta t$$

$$\omega_y = (b/a) \omega_{x0} \sin \beta t + \omega_{y0} \cos \beta t$$

^{*}For representative values of moments of inertia for Skylab B. See next section for the values used throughout this paper.

and

$$\beta = \sqrt{k_x k_y} \omega_0$$

The subscript zero on ω_{x0} and ω_{y0} refers to zero time. It is clear from (2) and (3) that $\bar{\omega}$ sweeps out a right elliptical cone. The tip of $\bar{\omega}$ moves along an ellipse whose ratio of major to minor axes, b/a , depends only on the system moments of inertia. The wobble frequency, β , is simply a constant times the spin frequency.

This analysis can be extended through the use of appropriate Euler angles to describe the motion in inertial coordinates. The extension is not carried forward here. An alternative method based on the Poincot construction is used instead since it is not limited by the above approximation and a single integration is required instead of three.

The above development is included here as the results compliment those to be presented next which will not give the time dependence.

Conservation Laws and Poincot Construction

The conservation laws, energy and angular momentum, constrain the Skylab system in rotation. As there are two constraints and three degrees of rotational freedom it is not surprising that the system moves, i.e., $\bar{\omega}$ is a function of time.

A variety of methods may be employed to determine the resulting motion. A convenient one which provides insight into the repetitive nature of the phenomena is based on the Poincot construction (1).

In this method, the inertia ellipsoid is made to roll without slipping on a plane normal to \bar{H} , the so-called invariable plane. Figure 1 shows the arrangement. The resulting loci of the points of contact on the ellipsoid and invariable plane are called the polhode and the herpolhode, respectively.

Figure 2 shows a plot of the projection of unit vectors in the direction of $\bar{\omega}$ and the plus Z axis on the invariable plane

for $\gamma = .0001$, $k_{xz} = 0.203$, and $k_{yz} = 0.963$.*,** The figure covers one half a revolution of the inertia ellipsoid. The pattern repeats for the next half revolution starting with new initial points existing at the end of the first half rotation. The motion in the invariable plane is not periodic since the patterns do not close.

The angles $\theta_{\omega h}$ and θ_{zh} are less than 10° for range of γ expected. It is possible therefore to interpret the radial coordinates of the curves directly in terms of these angles. To facilitate this reading, the radial coordinate is marked in degrees rather than radians.

It is easy to show that the curves in Figure 2 scale as $\sqrt{\gamma}$ while maintaining their form. These curves, with appropriate scaling, therefore completely describe the wobble state of Skylab in inertial coordinates for the given values of the parameters k_{xz} and k_{yz} .

Numerical Results

Computed values of the variables are given in Table I for $\gamma = .001$. This value of γ is six times as large as the maximum value obtainable due to the simultaneous displacement of three crew members.

Even for this relatively large value of γ , the variation in ω_z is seen to be quite small thus validating the assumption used in the approximate analysis.

The angles θ_{zh} and $\theta_{\omega h}$ can be compared with Figure 2. They stand approximately as $\sqrt{10} : 1$, thus confirming the $\sqrt{\alpha}$ dependence indicated earlier.

The increase in ϕ_{ω} and ϕ_z indicated in Table 1, which corresponds to a $1/4$ revolution of the inertia ellipsoid is only a trifle larger, approximately 2° , than for Figure 2. These quantities are therefore relatively independent of γ which is the basis of the assertion that Figure 2 preserves its shape and only scales in size as γ changes.

*Obtained from estimated values $I_x = 0.929 \times 10^6$ slug ft², $I_y = 4.405$ slug ft², and $I_z = 4.572 \times 10^6$ slug ft².

** $\gamma = .00015$ is largest value obtainable from simultaneous displacement of three, 6.6 slug astronauts.

Wobble Angle

It appears to be common to characterize wobble by referring to a wobble angle. There are three possible angles, $\theta_{\omega h}$, $\theta_{\omega z}$, and θ_{zh} and it is rare for the precise angle to be identified in such references. In any case, these angles are of limited utility in describing wobble since they vary in time. In the case described by Table I, θ_{zh} and $\theta_{\omega h}$ vary by nearly 10 : 1 and $\theta_{\omega z}$ varies by more than 2 : 1.

It is for this reason that γ , the excess energy ratio, is recommended for use in describing wobble. This quantity meets a desirable prerequisite for use as a parameter, i.e., it is invariant with time.

Checks

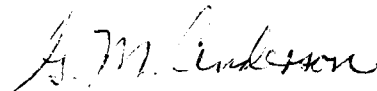
The computer program used for Figure 2 was checked in various ways. All purely algebraic manipulations were checked by slide rule methods. The length of the polhode can be approximated by a complete elliptic integral of the second kind.

Finally the curves of Figure 2 were compared with the results of L. E. Voelker's program which integrates the equations of motion using Euler angles.

All these checks on the program sustained the validity of the computer results.

Acknowledgement

Mrs. Patricia Dowling wrote the computer program. Her services are gratefully acknowledged.



G. M. Anderson

1022-GMA-cf

Attachments

Figures 1, 2
Table I
Appendix

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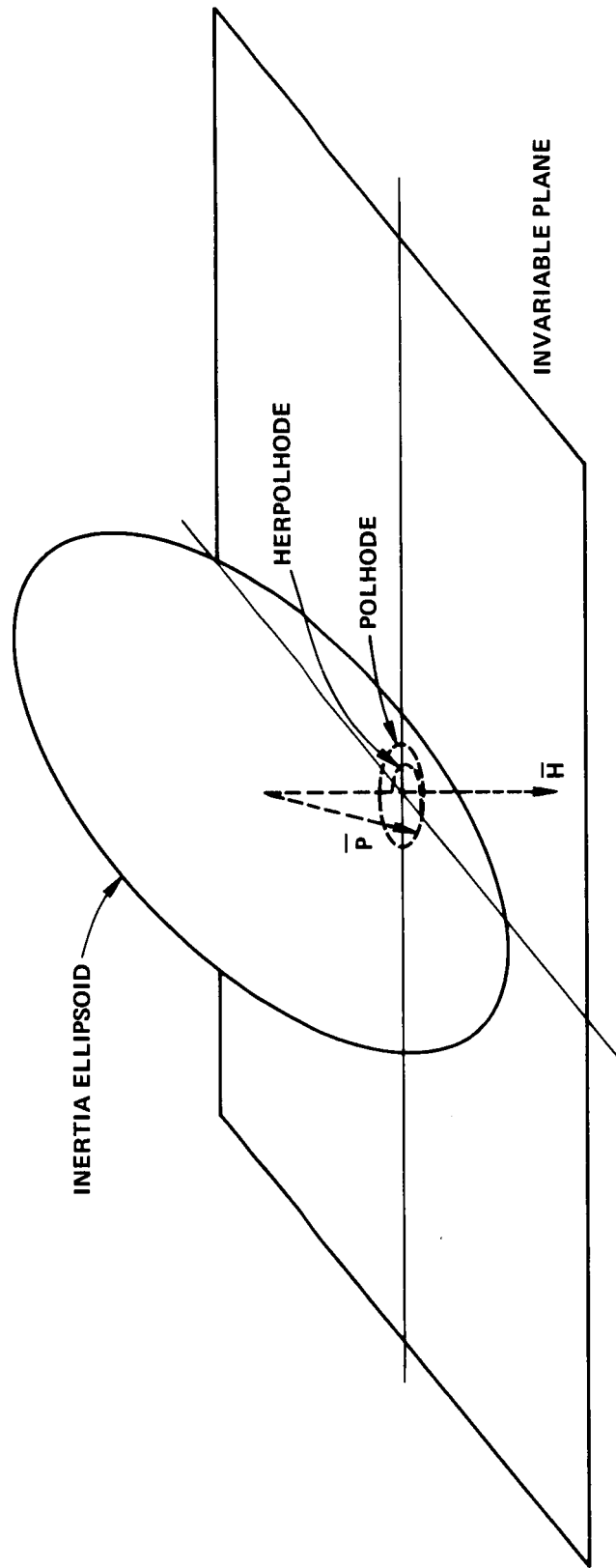


FIGURE 1 - MOTION OF INERTIA ELLIPSOID ON INVARIABLE PLANE

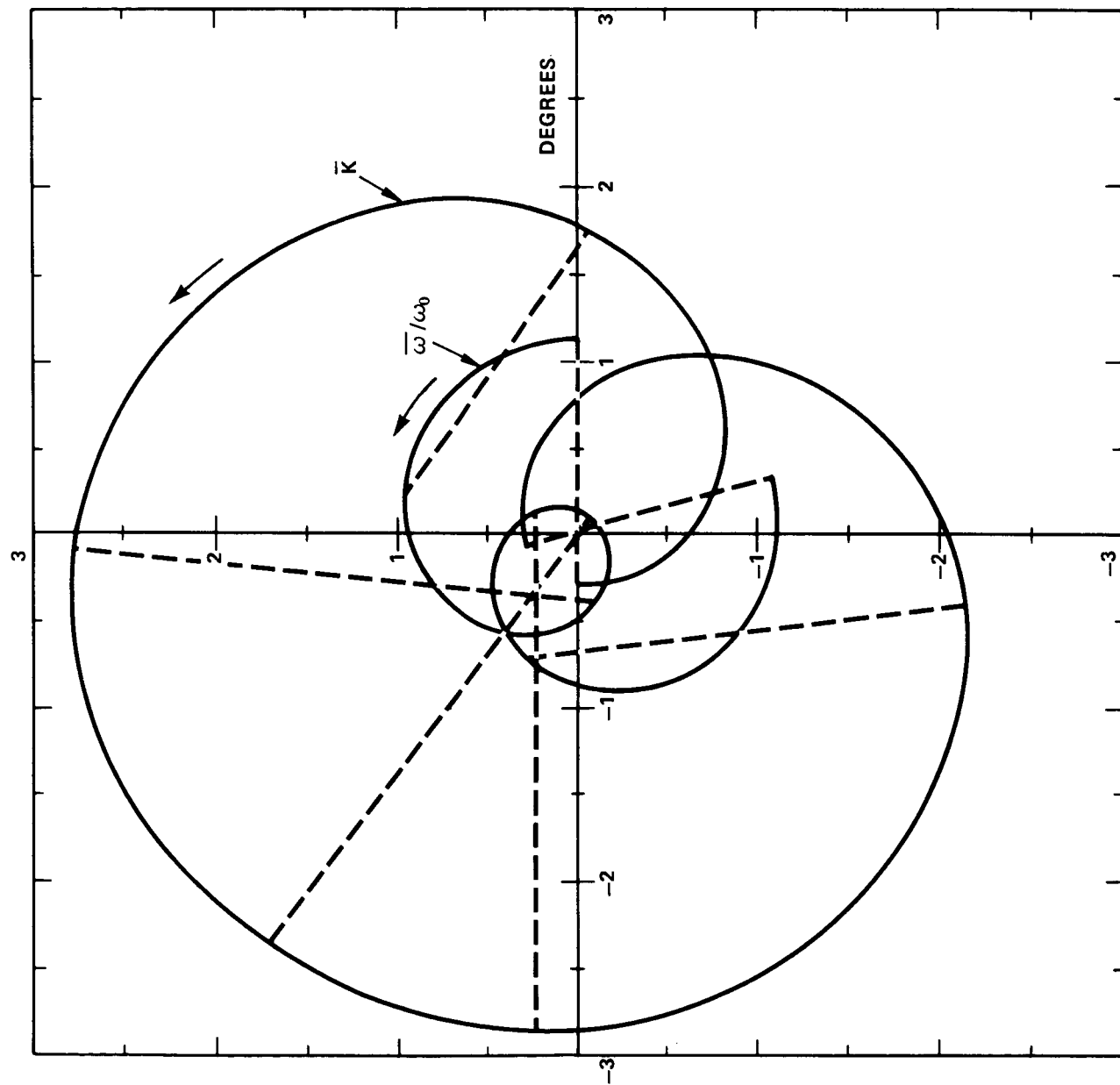


FIGURE 2 - PROJECTION OF $\bar{\omega}/\omega_0$ AND \bar{K} ON PLANE NORMAL TO \bar{H}

$\gamma = 0.0001$

DASHED LINES CONNECT CORRESPONDING POINTS.

TABLE I

u	0	.2	.4	.6	.8	1.0
ω_x/ω_o	.079	.077	.072	.063	.047	0
ω_y/ω_o	0	.034	.067	.101	.135	.168
ω_z/ω_o	.999	.999	.998	.995	.991	.987
ω/ω_o	1.003	1.003	1.003	1.002	1.002	1.001
ρ_x/ρ_o	.079	.077	.072	.063	.047	0
ρ_y/ρ_o	0	.034	.067	.101	.135	.168
ρ_z/ρ_o	.999	.999	.997	.995	.991	.986
ρ/ρ_o	1.002	1.002	1.002	1.002	1.001	1.001
$\theta_{zh}(\text{deg})$.915	2.06	3.82	5.64	7.48	9.34
$\theta_{wh}(\text{deg})$	3.59	3.51	3.28	2.87	2.17	.35
$\theta_{\omega z}(\text{deg})$	4.49	4.81	5.64	6.82	8.19	9.69
$\phi_{\omega}(\text{deg})$	0	31.0	63.6	100	146	325
$\phi_z(\text{deg})$	180	274	318	358	404	505

Conditions

$$\gamma = .001, k_{xz} = .203, k_{yz} = .963$$

$$u = \eta_y/b = (\omega_y/\omega_o)/b$$

APPENDIXNotation

\bar{H}	-	Angular momentum
T	-	Rotational kinetic energy
T_0	-	Minimum T for given \bar{H}
$\bar{\omega}$	-	Angular velocity
ω_0	-	$ \bar{\omega} $ for $T = T_0$
\bar{n}	=	$\bar{\omega}/\omega_0$
I	-	Moment of inertia in direction \bar{n}
x, y, z	-	Principal axes coordinate system
$I_z > I_y > I_x$	=	Principal axes moments of inertia
$\bar{\rho}$	=	\bar{n}/\sqrt{I} Locus of tip of $\bar{\rho}$ is inertia ellipsoid
ρ_0	=	$1/\sqrt{I_z}$
$\bar{\zeta}$	=	$\bar{\rho}/\rho_0$
$\theta_{\omega h}$	-	Angle between $\bar{\omega}$ and \bar{H}
$\theta_{\omega z}$	-	Angle between $\bar{\omega}$ and plus z axis
θ_{zh}	-	Angle between plus z axis and \bar{H}
γ	=	$\frac{T-T_0}{T_0}$ Excess energy ratio
k_{xz}	=	I_x/I_z
k_{yz}	=	I_y/I_z
k_x	=	$(I_z - I_y)/I_x$

$$\begin{aligned}
 k_y &= \frac{(I_z - I_x)}{I_y} \\
 k_z &= (I_y - I_x) / I_z \\
 \beta &= \sqrt{k_x k_y} \omega_0 \\
 a^2 &= \frac{\gamma}{k_{xz}(1 - k_{xz})} \\
 b^2 &= \frac{\gamma}{k_{yz}(1 - k_{yz})}
 \end{aligned}$$

Absence of a bar indicates the magnitude of a vector and subscripts x, y, or z refer to components along principal axes.

Conservation Laws and Poincot Construction

The conservation of energy and angular momentum yield

$$(a) \quad I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 = 2T$$

(A-1)

$$(b) \quad (I_x \omega_x)^2 + (I_y \omega_y)^2 + (I_z \omega_z)^2 = H$$

Dividing (A-1a) by $2T_0 = I_z \omega_0^2$ and (A-1b) by $H = I_z \omega_0$ gives

$$k_{xz} \eta_x^2 + k_{yz} \eta_y^2 + \eta_z^2 = \gamma + 1$$

(A-2)

$$k_{xz}^2 \eta_x^2 + k_{yz}^2 \eta_y^2 + \eta_z^2 = 1$$

For a given system, the parameter γ is seen to completely characterize the wobble state. It is easy to show that the range on γ is

$$(A-3) \quad 0 \leq \gamma \leq \frac{1}{k_{xz}} - 1$$

The condition $\gamma = 0$ is the wobble free state with

$$\omega_x = \omega_y = 0$$

$$\omega_z = \omega_o$$

The quantity η_z may be eliminated from Equations A-2 to yield

$$\frac{\eta_x^2}{a^2} + \frac{\eta_y^2}{b^2} = 1$$

The projection of the locus of the tip of $\bar{\omega}$ on the xy plane is an ellipse. The locus is not a plane figure in this instance since ω_z does vary somewhat.

The three wobble angles are readily determined and are given by the following formulas:

$$\theta_{zh} = \cos^{-1} \left(\eta_z / \sqrt{k_{xz}^2 \eta_x^2 + k_{yz}^2 \eta_y^2 + 1} \right) = \cos^{-1} \eta_z$$

$$(A-4) \quad \theta_{\omega z} = \cos^{-1} (\eta_z / \eta)$$

$$\theta_{\omega h} = \cos^{-1} (\gamma + 1) / \eta$$

the last relationship coming from

$$\bar{\omega} \cdot \bar{H} = 2T$$

A complete description of the behavior of the system in inertial coordinates requires an integration. The method used here is based on the Poinot construction.

The inertia ellipsoid of Figure 1 is the locus of a vector $\bar{\rho}$ given by

$$(A-5) \quad \bar{\rho} = \bar{n}/\sqrt{I}$$

where I is the moment of inertia in a direction \bar{n} .

Since

$$\rho = 1/\sqrt{I}$$

it follows that

$$(A-6) \quad (\omega/\rho)^2 = 2T$$

and therefore

$$(A-7) \quad (\omega_o/\rho_o)^2 = 2T_o$$

Dividing (A-6) by (A-7) gives

$$(\eta/\zeta)^2 = \gamma + 1$$

and

$$(A-8) \quad \bar{\zeta} = \bar{\eta} / \sqrt{\gamma+1}$$

The foregoing relationships are sufficient to formulate a differential equation describing the rolling of the inertia ellipsoid on the invariable plane.

Let

- ds_p - element of length on polhode
- ds_h - element of length on herpolhode
- $r_{h\omega}$ - radius vector in invariable plane from \bar{H} to tip of $\bar{\rho}$.
- ϕ_ω - angle between $r_{h\omega}$ and a reference line in invariable plane.

Then

$$\frac{r_{h\omega}}{\rho_o} = \left(\frac{\rho}{\rho_o} \right) \sin \theta_{\omega h} = \zeta \sin \theta_{\omega h}$$

$$(A-9) \quad \frac{ds_p}{\rho_o} = \sqrt{d\zeta_x^2 + d\zeta_y^2 + d\zeta_z^2}$$

$$\frac{ds_h}{\rho_o} = \sqrt{\left(\frac{dr_{h\omega}}{\rho_o} \right)^2 + \left(\frac{r_{h\omega}}{\rho_o} d\phi_\omega \right)^2} = \frac{ds_p}{\rho_o}$$

The last expression can be solved for $d\phi_\omega$. The associated integration was performed numerically on a digital computer for the curves given in Figure 2.

The location of the projection of \bar{k} , unit vector along Z, can be determined once ϕ_ω is known. It is easy to show that

$$\cos \phi_z = \frac{\cos \theta_{zh} \cos \theta_{\omega h} - \cos \theta_{\omega z}}{\sin \theta_{zh} \sin \theta_{\omega h}}$$

where ϕ_z is measured at origin between line of angle $\phi_\omega + \pi$ and projection of \bar{k} .

Although the curves of Figure 2 were computed using $\bar{\rho}/\rho_o$ the results apply to $\bar{\omega}/\omega_o$ since these vectors are parallel.

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